Note Title 2/14/2019

$x[n] \longrightarrow$ [syst.] $\longrightarrow y[n]$

$\delta[n] \longrightarrow$ [syst.] $\longrightarrow h[n]$ unit-pulse response

$x[n] \longrightarrow$ [syst.] $\longrightarrow y[n] = h[n] * x[n]$

$y[n] = y[n-1] + 2x[n] - 3x[n-2]$ ← system

If $x[n] = u[n]$, find $y[n]$

① $h[n] = h[n-1] + 2\delta[n] - 3\delta[n-2]$

solve for $h[n]$

$h[0] = h[-1] + 2\delta[0] - 3\delta[-2] = 2$

(0 → 0)

$h[1] = h[0] + 2\delta[1] - 3\delta[-1] = 2$

$\vdots \quad \vdots \quad 0 \quad 0 \quad \vdots$

$h[n]$

② $y[n] = x[n] * h[n]$ System must be LTI

$x(t) \longrightarrow$ [syst.] $\longrightarrow y(t)$

$y(t) = x(t) * h(t)$ | $h(t) = y(t)$ | $x(t) = \delta(t)$

Example: find the convolution of $x(t)=e^{t}u(t)$ and $v(t)=e^{-t}u(t)$, where a and b are nonzero constant numbers and $u(t)$ is the unit step function:

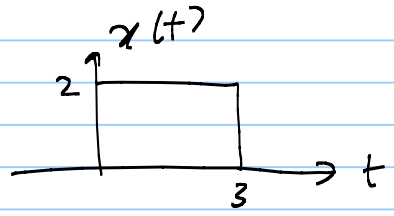
$$x(t) * v(t) = \int_0^t x(\lambda)\, v(t-\lambda)\, d\lambda$$

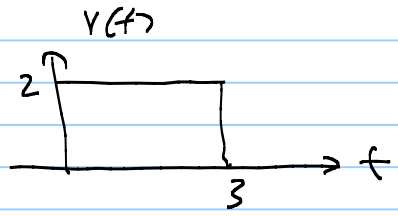
$$= \int_0^t e^{\lambda} \cdot e^{-(t-\lambda)}\, d\lambda$$

$$= \int_0^t e^{-t} e^{2\lambda}\, d\lambda = e^{-t} \int_0^t e^{2\lambda}\, d\lambda$$

$$= e^{-t}\left[\frac{1}{2} e^{2\lambda}\right]_0^t = \frac{e^{-t}}{2}\left[e^{2t} - 1\right]$$

$$= \frac{1}{2}\left[e^{t} - e^{-t}\right]$$

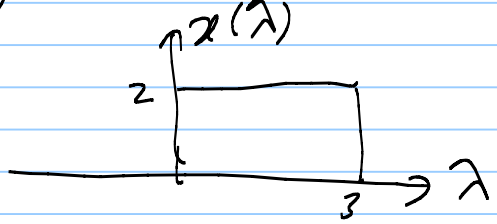


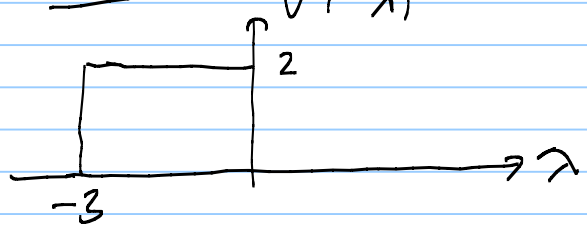


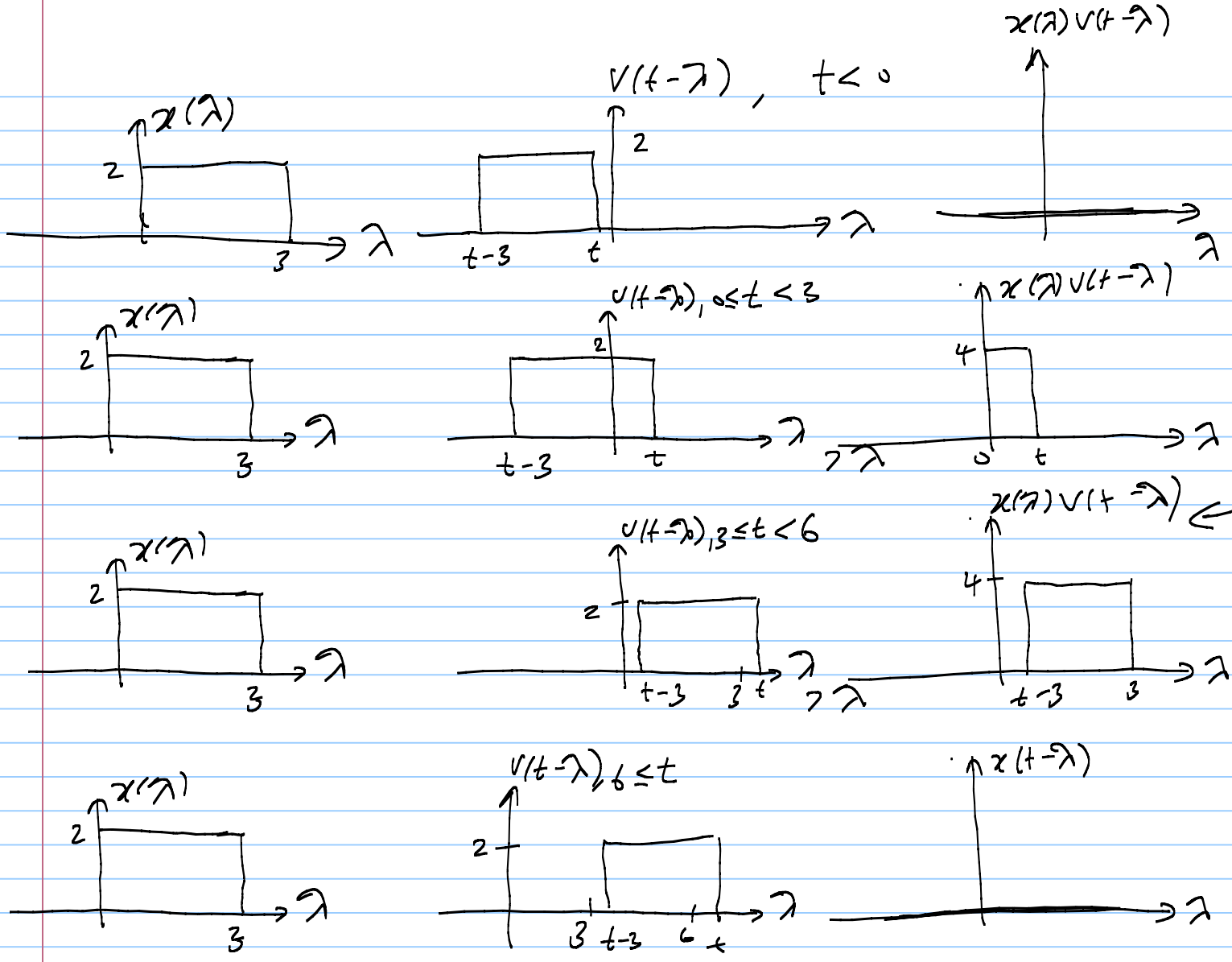
$x(t) * v(t)$?

$$x(t) * v(t) = \int_0^t x(\lambda)\, v(t-\lambda)\, d\lambda$$

(1)







for $t<0$ & $t\geq 6$, $x(t)*v(t)=0$

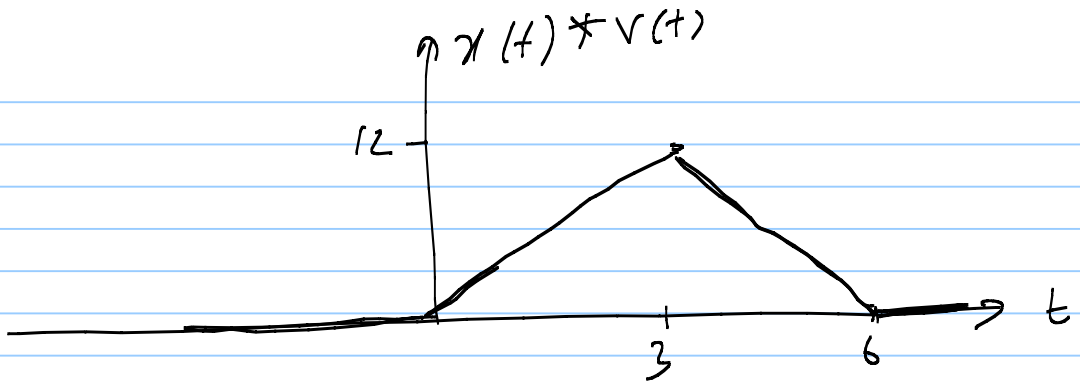
* for $0\leq t<3$

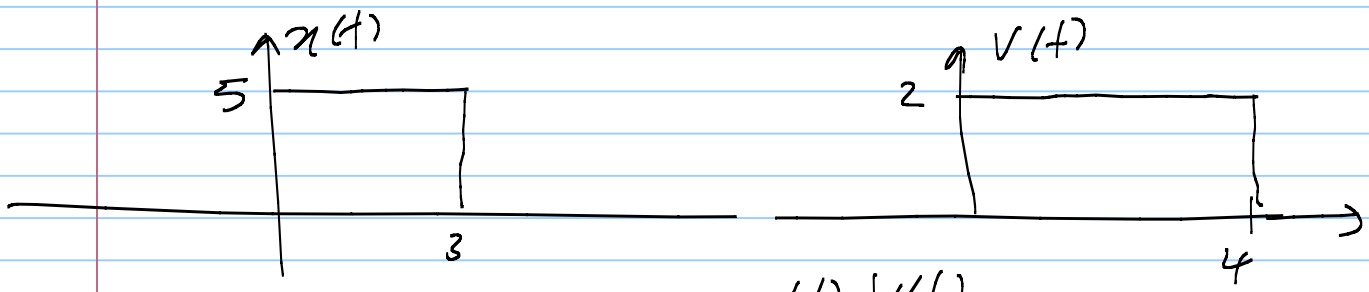
$$x(t)*v(t)=\int_0^t 4\,d\lambda = 4\lambda\Big|_0^t = 4t$$

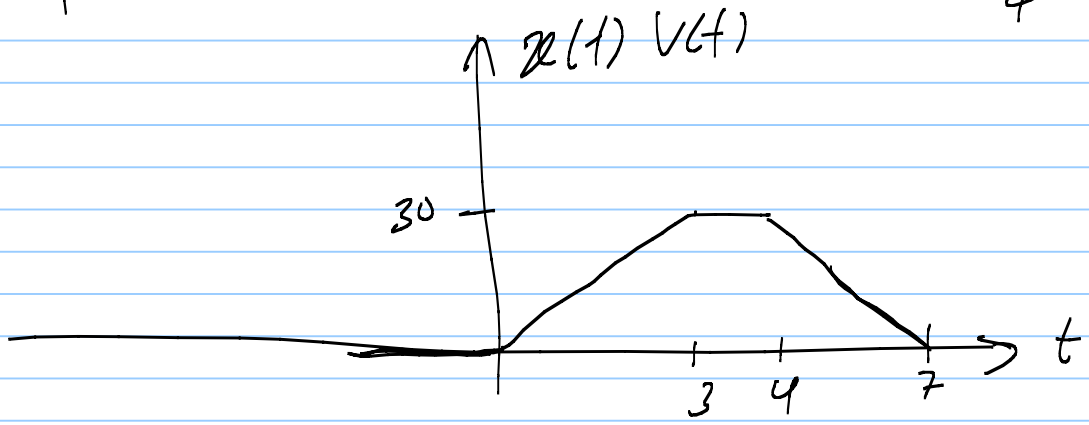
* for $3\leq t<6$

$$x(t)*v(t)=\int_{t-3}^{3} 4\,d\lambda = 4\lambda\Big|_{t-3}^{3} = 4\left[3-t+3\right]$$

$$= -4t+24$$


x(t) * v(t)
12
3
6
t


x(t)
5
3
v(t)
2
4


x(t) v(t)
30
3
4
7
t